



Trinity College

Semester Two Examination, 2017

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 3 AND 4

Section One:
Calculator-free

If required by your examination administrator, please
place your student identification label in this box

Student Number: In figures

--	--	--	--	--	--	--	--	--

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (52 Marks)

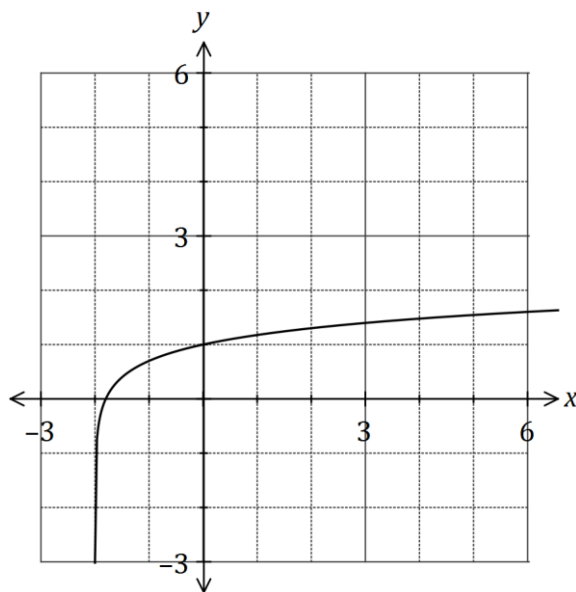
This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(5 marks)

The graph of $y = f(x)$ is shown below, where $f(x) = \log(5x + 10)$.



- (a) Explain why y is a function of x over the natural domain of f . (1 mark)

- (b) Determine a function for the inverse of f . (2 marks)

- (c) On the axes above, sketch the graph of the inverse of f . (2 marks)

Question 2

(7 marks)

- (a) Given that $z - 1 + i$ is a factor of $z^4 + az^2 + bz$, determine the values of the real constants a and b . (4 marks)

- (b) Solve the equation $z^4 - z^2 - 20 = 0$. (3 marks)

Question 3

(7 marks)

Let $u = \sqrt{3} - i$ and $z = 2 + 2i$.

(a) Determine the argument of $z \div u^3$.

(3 marks)

(b) Determine the real part of $(iz)^4$.

(4 marks)

Question 4

(7 marks)

- (a) Determine the slope of the graph of $2x^2 + y^2 = 3xy$ at the point $(1, 2)$. (3 marks)

- (b) Use the substitution $u = 5x^2 - 4$ to express $\int_1^2 \frac{10x}{\sqrt{5x^2 - 4}} dx$ in terms of u and hence evaluate the integral. (4 marks)

Question 5

(6 marks)

Two functions are defined by

$$f(x) = \frac{x-2}{x^2-4} \quad \text{and} \quad g(x) = x+7.$$

(a) Determine $g \circ f(1)$.

(1 mark)

(b) Determine a simplified expression for $f \circ g(x)$ and state the domain and range for this composite function.

(5 marks)

Question 6

(5 marks)

A plane passes through the points $A(0, 0, 1)$, $B(1, -2, 3)$ and $C(3, 1, -1)$. Determine the Cartesian equation of the plane.

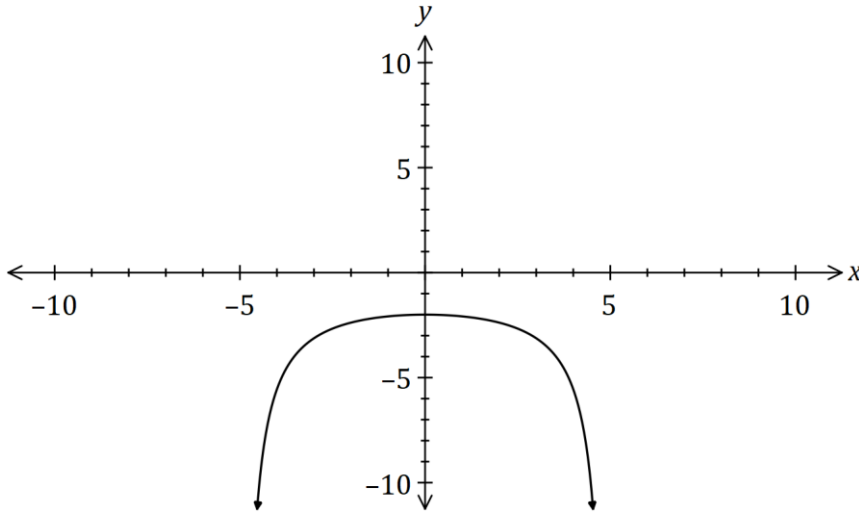
Question 7

(7 marks)

- (a) Part of the graph with equation $y = \frac{50}{x^2 - 25}$ is shown below.

Complete the sketch, clearly indicating the location of any asymptotes.

(2 marks)



- (b) Determine the area bounded by $y = \frac{50}{x^2 - 25}$, the x -axis, the y -axis and the line $x = 4$.

(5 marks)

Question 8

(8 marks)

- (a) Determine all solutions to the equation $2^7 z^6 = -\sqrt{2} + \sqrt{2}i$ in the form $r \operatorname{cis} \theta$ where $r > 0$ and $-\pi < \theta \leq \pi$. (5 marks)

- (b) If w is any complex cube root of unity, simplify $(1 + 4w)(1 + 4w^2)$. (3 marks)

Additional working space

Question number: _____

